from a combination of nonlinear inviscid phenomena together with significant induced effects resulting from oscillatory transition point movement.

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Lifting and Nonlifting Kernel Functions for Cascade and Isolated Airfoils

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THERE exists a considerable amount of similarity between the theoretical analyses of unsteady aerodynamics of cascade airfoils and isolated airfoils. The lifting and nonlifting kernel functions that relate the surface upwash distribution to the surface pressure load are of fundamental importance in linearized, unsteady, subsonic, potential-flow aerodynamics. It is the purpose of this Note to derive these kernel functions for two-dimensional flows in a unified fashion via the Fourier transform technique and specifically discuss the singular behavior of the nonlifting kernel function in the limit of zero Mach number. Because of its physical significance, the traveling wave-type of motion of the cascade blades will be considered.

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Lifting Kernel Function

The subsonic lifting kernel function for two-dimensional linear cascade airfoils was evaluated previously by Fleeter[1] and Kaji and Okazaki.[2] Fleeter indicated the singular behavior of the lifting kernel function and computed the kernel function by inverting its Fourier transform numerically. Kaji and Okazaki derived an analytical expression for the lifting kernel function using the doublet singularity method. Their expression, as it stands, is inappropriate for zero Mach number flows. In this paper, a different expression for the lifting kernel is derived by analytically inverting Fleeter's Fourier integral. The resulting expression is valid for all subsonic Mach numbers, including zero.

Following Fleeter,[1] the lifting kernel function $K(x)$ for harmonic motions with frequency ω in a uniform stream with Mach number M is the Fourier inversion of K^*, i.e.,

$$K(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}K^*e^{i\alpha x}\mathrm{d}\alpha \tag{1}$$

where

$$K^*=\frac{i}{2}\cdot\frac{\mu\sin(\mu h)}{[\alpha+(\omega/U)][\cos(\mu h)-\cos(\alpha d-\sigma)]} \tag{2}$$

and

$$\mu^2=\beta^2\left[\left(\frac{\omega M}{U\beta^2}\right)^2-\left(\alpha-\frac{\omega M^2}{U\beta^2}\right)^2\right]$$

In the preceding equation, σ is the interblade phase angle and $\beta=\sqrt{1-M^2}$. Figure 1 defines the cascade geometric parameters d and h. It is noted that the expression of K^* is for any arbitrary stagger angle, and Fleeter's expression[1] is for zero stagger angle only. Physically, K is the upwash velocity corresponding to a spatial impulse pressure differential across airfoil upper and lower surfaces.

To simplify the Fourier inversion,

$$\bar\alpha=\alpha-\frac{\omega}{U}\frac{M^2}{\beta^2}$$

is defined. Then from Eq. (1),

$$K(x)=\frac{i}{4\pi}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]$$

$$\times\int_{-\infty}^{\infty}\frac{\mu\sin(\mu h)e^{i\bar\alpha x}\mathrm{d}\bar\alpha}{[\bar\alpha+(\omega/\beta^2U)][\cos(\mu h)-\cos(\bar\alpha d-\sigma')]} \tag{3}$$

where

$$\sigma'=\sigma-\frac{M^2}{\beta^2}\cdot\frac{\omega d}{U}$$

and

$$\mu^2=-\beta^2\left[(\bar\alpha^2)-\left(\frac{\omega M}{U\beta^2}\right)^2\right] \tag{4}$$

The poles of the integrand in the complex $\bar\alpha$ plane are all of first-order and are located at

$$-\frac{\omega}{U}\frac{1}{\beta^2}\quad\text{and}\quad\bar\alpha_n^{\pm}$$

where $\bar\alpha_n^{\pm}$ are the zeros of $[\cos(\mu h)-\cos(\bar\alpha d-\sigma')]$, and the superscripts $\pm$ refer to the upper (+) and lower (−) $\bar\alpha$ half-

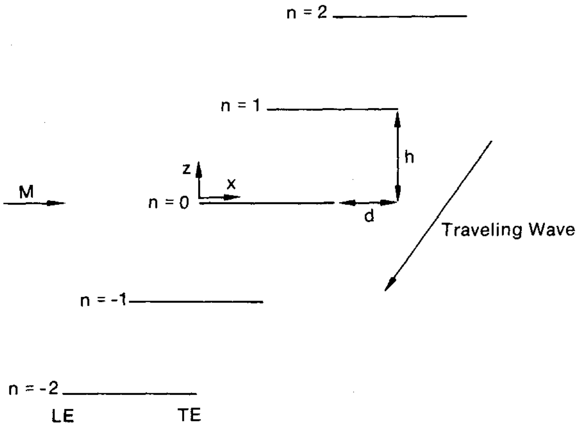

Fig. 1 Cascade configuration.

planes. The poles $\bar{\alpha}^{\pm}$ are found to be

$$\bar{\alpha}=\bar{\alpha}_{n_R}\pm i\bar{\alpha}_{n_I}$$

where

$$\bar{\alpha}_{n_R}=\frac{-d(2n\pi-\sigma')}{\beta^2h^2+d^2},\qquad \bar{\alpha}_{n_I}=\frac{hR_n}{\beta^2h^2+d^2}$$

and

$$R_n=\left[\beta^2(2n\pi-\sigma')^2-\left(\frac{\omega}{U}\frac{M}{\beta}\right)^2(\beta^2h^2+d^2)\right]^{1/2} \tag{5}$$

The square root in Eq. (5) has to be taken properly. For subresonant cases,

$$\left(\frac{\omega}{U}\frac{M}{\beta}\right)^2(\beta^2h^2+d^2)<\beta^2(2n\pi-\sigma')^2$$

and R_n is positive and real. For superresonant cases,

$$\left(\frac{\omega}{U}\frac{M}{\beta}\right)^2(\beta^2h^2+d^2)>\beta^2(2n\pi-\sigma')^2$$

and R_n is taken as purely imaginary and positive. This is so taken because 1) μ is chosen so that it has a positive addition part; and 2) ω has a small negative imaginary part, as is generally done in unsteady aerodynamic theories. See Ref. 4 for further details.

Since ω is assumed to have a small negative imaginary part, it is easy to recognize that $-\omega/(U\beta^2)$ and $\bar{\alpha}_n^+$ lie in the upper half-$\bar{\alpha}$ plane and $\bar{\alpha}_n^-$ lie in the lower half-$\bar{\alpha}$ plane. Now, Eq. (3) can be evaluated analytically by using the Residue Theorem. To enclose the contour in the $\bar{\alpha}$ plane, the semicircles are extended to infinity, the upper semicircle for $x>0$, and the lower semicircle for $x<0$, are chosen. This choice will ensure that the contribution of the semicircles to the integral vanishes as their radii approach infinity. This is true because 1) the part of the integrand that multiplies $e^{i\bar{\alpha}x}$ is of order one when $|\alpha|$ approaches infinity, and 2) Jordan's Lemma can be extended to apply to the present case by simple differentiation. Therefore,

$$K(x)=-\frac{\mathrm{sgn}(x)}{2}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]\sum\text{residues}$$

where Σ residues indicate the sum of the residues corresponding to the simple poles. For $x>0$ only, the poles in the upper half-plane are considered and for $x<0$, the lower plane. The result is as follows. $K(x)$ is singular at $x=0$. Extracting out the singular terms,

$$K(x)=K_s(x)+K_c(x) \tag{6}$$

where

$$K_s(x)=\frac{\beta}{2\pi x}-\frac{i\omega}{2\pi\beta U}\ln\left|\frac{\pi x}{2\beta h}\right|$$

$$K_c(x)=\frac{\dfrac{\omega}{4U}\sinh\left(\dfrac{\omega h}{U}\right)[1+\mathrm{sgn}(x)]\exp\left[-i\dfrac{\omega x}{U}\right]}{\cosh\left(\dfrac{\omega h}{U}\right)-\cos\left(\dfrac{\omega d}{U}+\sigma\right)}$$

$$+\frac{i}{2h}\left[\sum_{n=0,\pm1,\pm2,\ldots}k_n(x)\right]+\frac{1}{h}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]$$

$$\times\left[(\mathrm{sgn}x)S_1+i\frac{\omega h}{U}S_2\right]+\frac{1}{2h}\left\{\left[\mathrm{csch}\left(\frac{\pi x}{\beta h}\right)-\frac{\beta h}{\pi x}\right]\right.$$

$$\left.+\left(\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]-1\right)\mathrm{cseh}\left(\frac{\pi x}{\beta h}\right)\right\}-\frac{i\omega}{2\pi\beta U}$$

$$\times\left[\ln\left(\frac{\tanh(\pi x/2\beta h)}{(\pi x/2\beta h)}\right)+\left(\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]-1\right)\ln\tanh\left|\frac{\pi x}{2\beta h}\right|\right]$$

$$k_n(x)=\frac{(\mu_nh)^2e^{i\alpha_nx}}{\left(\alpha_nh+\dfrac{\omega h}{U}\right)R_n}$$

$$-\frac{\pi^2\beta^2(2n-1)^2\exp\left[i\dfrac{M^2}{\beta^2}\dfrac{\omega x}{U}-\dfrac{R_n^\pi}{\beta^2}\dfrac{|x|}{h}\right]}{\left(\dfrac{\omega h}{U}+iR_n^\pi\mathrm{sgn}x\right)R_n^\pi}$$

$$S_1=\sum_{n=1}^{\infty}\left\{\frac{\pi^2\beta^2(2n-1)^2\exp\left[-\dfrac{R_n^\pi}{\beta^2}\dfrac{|x|}{h}\right]}{\left(\dfrac{\omega h}{U}\right)^2+R_n^{\pi2}}\right.$$

$$\left.-\exp\left[-(2n-1)\frac{\pi}{\beta}\frac{|x|}{h}\right]\right\}$$

$$S_2=\sum_{n=1}^{\infty}\left\{\frac{\pi^2\beta^2(2n-1)^2\exp\left[-\dfrac{R_n^\pi}{\beta^2}\dfrac{|x|}{h}\right]}{\left[\left(\dfrac{\omega h}{U}\right)^2+R_n^{\pi2}\right]R_n^\pi}\right.$$

$$\left.-\frac{\exp\left[-(2n-1)\dfrac{\pi}{\beta}\dfrac{|x|}{h}\right]}{\pi\beta(2n-1)}\right\}$$

$$R_n=\left[\beta^2(2n\pi-\sigma')^2-\left(\frac{M}{\beta}\frac{\omega}{U}\right)^2(d^2+\beta^2h^2)\right]^{1/2}$$

$$R_n^\pi=\left[\beta^2\pi^2(2n-1)^2-\left(M\frac{\omega h}{U}\right)^2\right]^{1/2}$$

$$\sigma' = \sigma - \frac{M^2}{\beta^2}\frac{\omega d}{U}, \quad \alpha_n = \alpha_{n_R} + i\alpha_{n_I}\mathrm{sgn}(x)$$

$$\alpha_{n_R} = \frac{M^2h^2(\omega/U) - (2n\pi - \sigma)d}{\beta^2h^2 + d^2}, \quad \alpha_{n_I} = \frac{hR_n}{\beta^2h^2 + d^2}$$

$$\mu_n h = a_n + id\alpha_{n_I}\mathrm{sgn}(x), \quad a_n = \frac{2n\pi - \sigma'}{1 + (d^2/\beta^2h^2)}$$

it should be noted that

$$R_n = \left|\beta^2(2n\pi - \sigma')^2 - \left(\frac{M}{\beta}\frac{\omega}{U}\right)^2(d^2 + \beta^2h^2)\right|^{1/2}$$

when

$$\beta^2(2n\pi - \sigma')^2 > \left(\frac{M}{\beta}\frac{\omega}{U}\right)^2(d^2 + \beta^2h^2)$$

and

$$R_n = i\left|\beta^2(2n\pi - \sigma')^2 - \left(\frac{M}{\beta}\frac{\omega}{U}\right)^2(d^2 + \beta^2h^2)\right|^{1/2}$$

when

$$\beta^2(2n\pi - \sigma')^2 < \left(\frac{M}{\beta}\frac{\omega}{U}\right)^2(d^2 + \beta^2h^2)$$

A similar definition holds for R_n^π.

Resonance occurs when $R_n = 0$, i.e.,

$$\frac{M}{\beta}\frac{\omega}{U} = \pm\frac{\beta(2n\pi - \sigma')}{(\beta^2h^2 + d^2)^{1/2}} \qquad n = 0, \pm 1, \pm 2, \ldots$$

or more explicitly

$$\frac{\omega}{U} = \frac{2n\pi - \sigma}{d^2 + h^2}\left(d \pm \frac{1}{M}\sqrt{d^2 + \beta^2h^2}\right)$$

When $\sigma = 180$ deg and $d = 0$, the kernel function K reduces to the lifting kernel for soild wall wind-tunnel flow.[3] When $h \to \infty$, the familiar Possio kernel for isolated wings is recovered.

Nonlifting Kernel Function

The nonlifting kernel function $A(x)$ is the Fourier inversion of A^* which is defined as

$$A^* = \frac{i(\alpha + \omega/U)\sin(\mu h)}{2\mu[\cos(\mu h) - \cos(\alpha d - \sigma)]} \tag{7}$$

in Ref. 4. Physically, the nonlifting kernel function $A(x)$ is the pressure load due to airfoil-thickness-induced upwash. For the isolated wing case, A^* is inversely proportional to K^*, as defined in Eq. (2). The simple poles in Eq. (7) are the same as those of K^* except that $\alpha = -\omega/U$ is no longer a pole. A similar Fourier inversion process results in the following expression for $A(x)$:

$$A(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A^* e^{i\alpha x}d\alpha = A_s(x) + A_c(x) \tag{8}$$

where

$$A_s(x) = \frac{-1}{2\pi\beta x} - \frac{i}{2\pi\beta^3}\frac{\omega}{U}\ln\left|\frac{\pi x}{2\beta h}\right|$$

$$A_c(x) = \frac{i}{2}\left[\sum_{n=0,\pm 1,\pm 2,\ldots} M_n(x)\right] - \frac{1}{h\beta^2}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]$$

$$\times\left[T_1\mathrm{sgn}(x) - i\frac{\omega h}{U}T_2\right] + \frac{1}{2h\beta^2}\left\{\left[\frac{\beta h}{\pi x} - \mathrm{csch}\left(\frac{\pi x}{\beta h}\right)\right]\right.$$

$$\left. + \left(1 - \exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]\right)\mathrm{csch}\left(\frac{\pi x}{\beta h}\right)\right\} + \frac{i}{2\pi\beta^3}\frac{\omega}{U}$$

$$\times\left[\ln\left(\frac{\dfrac{\pi x}{2\beta h}}{\tanh\dfrac{\pi x}{2\beta h}}\right) + \left(1 - \exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]\right)\ln\tanh\left|\frac{x\pi}{\beta h}\right|\right]$$

$$M_n(x) = \frac{\alpha_n + (\omega/U)}{R_n}e^{i\alpha_n x} - \left[\frac{\omega h/U}{R_n^\pi} + i\mathrm{sgn}(x)\right]$$

$$\times\left\{\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U} - \frac{R_n^\pi}{\beta^2}\frac{|x|}{h}\right]\Big/h\beta^2\right\}$$

$$T_1 = \sum_{n=1}^{\infty}\left(\exp\left[-\frac{R_n^\pi}{\beta^2}\frac{|x|}{h}\right] - \exp\left[-(2n-1)\frac{\pi}{\beta}\frac{|x|}{h}\right]\right)$$

$$T_2 = \sum_{n=1}^{\infty}\left(\frac{\exp\left[-\dfrac{R_n^\pi}{\beta^2}\dfrac{|x|}{h}\right]}{R_n^\pi} - \frac{1}{\pi\beta}\frac{\exp\left[-(2n-1)\dfrac{\pi}{\beta}\dfrac{|x|}{h}\right]}{2n-1}\right)$$

Several special cases are:

1) When $\omega = 0, d = 0, \sigma = 0,$ $\quad A = \dfrac{-1}{2h\beta^2}\coth\dfrac{\pi x}{\beta h}$

2) When $\omega = 0, d = 0, \sigma = \pi/2,$ $\quad A = \dfrac{-1}{4h\beta^2}\mathrm{csch}\dfrac{\pi x}{2\beta h}$

3) When $\omega = 0, d = 0, \sigma = \pi,$ $\quad A = \dfrac{-1}{2h\beta^2}\mathrm{csch}\dfrac{\pi x}{\beta h}$

4) When $\omega = 0, h = \infty,$ $\quad A = \dfrac{-1}{2\pi\beta x}$

5) When $h = \infty,$

$$A(x) = \frac{(\omega/U)}{4\beta^3}\exp\left[i\frac{M^2}{\beta^2}\frac{\omega x}{U}\right]\left\{H_0^{(2)}\left(\left|\frac{M}{\beta^2}\frac{\omega x}{U}\right|\right)\right.$$

$$\left. + iM[\mathrm{sgn}(x)]H_1^{(2)}\left(\frac{M}{\beta^2}\frac{\omega x}{U}\right)\right\} \tag{9}$$

The nonlifting kernel function $A(x)$ becomes logarithmically singular when the Mach number asymptotically approaches zero, unless the frequency ω is exactly zero. This can be easily seen in Eq. (9) for isolated wings since $H_0^{(2)}(z) \sim \ln(z)$ as $z \to 0$. This difficulty is usually avoided by using small but nonzero Mach numbers in incompressible flow problems[5,6] or using the steady flow result, which is finite, in the context of quasi-steady analysis.[7]

Concluding Remarks

The lifting and nonlifting kernel funcions for two-dimensional subsonic cascade flows are systematically derived in forms for easy computation. Their relations to previous derived cascade results and isolated wing results are discussed.

The lifting kernel function is of central importance in the determination of aerodynamic forces due to airfoil oscillations, wing gust responses, and engine inlet distortions. The nonlifting kernel function is of primary interest when 1) flow separations are involved for both cascade and isolated wing oscillations,[4] and 2) the panel flutter problem is considered for the external type of flow.[5-7]

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J80-241

Modeling the Plasma Near-Wakes

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I. Introduction

WHEN a flat-based projectile moves rapidly in a gaseous medium, a cavity is momentarily created behind the moving body. The ambient gas tends to fill the cavity at the speed of its thermal motion. If the ambient gas is ionized as in the case of an ionosphere, the ambient electrons, having a greater mobility, implode into the empty space with the ions left behind, thus creating an electric field of charge separation. The field, so induced, will decelerate the advancing electrons very rapidly and accelerate the heavy ions slightly until a quasiequilibrium state is established, whereby they implode into the central core with a common speed of flow which is often designated as the speed of the wavefront. In addition to the fact that the plasma near-wake phenomenon is of intrinsic physical interest by itself, the importance of its relevancy to the in situ plasma measurements onboard the spacecraft is noteworthy.[1-4]

In order to sharpen our focus on the essential physics of the near-wakes in a tenuous plasma, consider an ambient, unmagnetized plasma in which a flat-based projectile of radius R and zero surface potential moves along its axis of symmetry at a constant mesothermal speed; to wit, the body speed V_b lies intermediate between the thermal speeds of the undisturbed ambient ions and electrons such that the ion speed $(\sqrt{2kT_i/m_i}) \ll V_b \ll$ the electron speed $(\sqrt{2kT_e/m_e})$. It is also assumed that the Debye length λ_D of the ambient plasma be small compared with the characteristic size of the body R. Both of the above-assumed conditions represent well a typical space plasma wake. The contemporary theories of plasma near-wakes[1] usually proceed to iterate for an approximate self-consistent solution of the particle and field distributions in the wake using the Vlasov equation for the particles and the Poisson equation for the field. The computations in such a numerical iteration are unusually heavy and often plagued with computational instabilities if, for example, the grid sizes used in the computations are not appropriately chosen. A satisfactory self-consistent theory of plasma near-wakes is still not in sight. The experimental results of plasma wakes in the plasma wind tunnels under the strict conditions of mesothermal speeds and large bodies $(\gg \lambda_D)$ are still scanty.[5,6] In view of the inherently nonstationary nature of the near-wake in free flight at a constant mesothermal body speed, special discretion is advised of its simulation in a conventional plasma wind tunnel. In essence, the status of our present understanding of the physics of plasma near-wakes can be considered rudimentary only. Furthermore, the above-mentioned kinetic theories of near-wakes have not taken into account the likelihood of plasma instabilities in the wake flow. The prospect of extending them to the turbulent near-wake following the kinetic approach is very dim indeed considering the present state-of-the-art.

II. Analogy to Near-Wakes

The purpose of this Note is to consider an alternative conceptual view of the near-wake phenomenon: the plasma cavity-filling process in developing a near-wake, under the mesothermal flow conditions, can be likened to the transient flow associated with a radially imploding cylindrical shock seen immediately after the rupture of a cylindrical diaphragm of radius R, which separates a empty cylindrical chamber from the ambient plasma, by an observer moving with the velocity of the projectile (of radius R). It is noted that, under the mesothermal flow condition, the plasma expansion into the near-wake develops on a time scale that is long compared with the electron plasma relaxation time; consequently, the ions move in a self-consistent electric field with which the electrons are already in Boltzmann equilibrium. In view of the fact that plasma is continuously being replenished from the ambient, it is assumed that the electron temperature T_e remains constant and is much higher than the ion temperature T_i in the plasma. The cold ion approximation $(T_i \ll T_e)$, which justifies the domination of induced field effect over the pressure gradient effect on ion motion, is used in the following simplifed fluid approach. This bithermal nature of the upper ionosphere was previously discussed.[1]

It is important to the present modeling of the plasma near-wakes to note that the time constant of the cavity formation, which is inversely proportional to the projectile speed V_b, is much smaller than that of the implosive plasma flow; the latter is of the same order as the ion thermal motion. Hence, it is stipulated that the ambient plasma at the leading edge of the initial cavity is primarily responsible for populating the near-wake; the plasma ions at the trailing edge of the cavity, on the other hand, do not contribute significantly because of the following condition: $\sqrt{2kT_i/m_i} \ll V_b$. According to the present modeling, in the near-wake behind a projectile moving at constant speed V_b along the axial z axis, the radial and axial distributions of ion density $n_i(r,z)$ and field $\phi(r,z)$ can be determined from their respective counterparts; namely, the radial and time-dependent $n_i(r,t)$ and $\phi(r,t)$ of a cylindrical imploding plasma flow, which starts at $t=0$ from

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